# Appendix B Dealing with zero or negative output

Although much of the literature makes assumptions that all firms are profitable following entry, this article allows for firms to produce zero output with their installed base remaining compatible with relevant networks. Following acquisition or entry, if equilibrium output is negative for any firm, the firm's output is reset to zero and equilibrium is redetermined for remaining firms. If the equilibrium calculates that more than one firm produces negative output, it is assumed that zero output and the redetermined equilibrium occurs hierarchically based on output, i.e. the firm calculated with the most negative output in the initial equilibrium is reset to zero first with equilibrium redetermined for remaining firms. Of course other factors may determine this zero output in equilibrium, but this algorithm is used for simplicity. This appendix describes how the aggregate output is calculated in a market with two incumbents and one entrant, if one or more firms has zero output.

## Compatibility

### Entry

If a firm makes negative output as a result of entry, inverse demand for the two remaining firms is given by:

$$p_i = A_i + \alpha_i \left( s_i + s_j + \sum_{k=1}^{n} b_k \right) - q_i - q_j \tag{1}$$

and profit from new customers is given by:

$$\pi_i = (p_i - c)\, q_i = \left( A_i + \alpha_i \left( s_i + s_j + \sum_{k=1}^{n} b_k \right) - q_i - q_j - c \right) q_i \tag{1a}$$

Using the first order condition for profit maximization and correct consumer expectations for market share ($s_j = q_i$), each remaining firm's output in Cournot competition is given by:

$$q_i = A_i + (\alpha_i - 1)\,(q_i + q_j) + \alpha_i \sum_{k=1}^{n} b_k - c \tag{1b}$$

Substituting output into profit:

$$\pi_i = (p_i - c)\, q_i = \left( A_i + (\alpha_i - 1)\,(q_i + q_j) + \alpha_i \sum_{k=1}^{n} b_k - c \right) q_i = q_i^2 \tag{1c}$$

finds profits are still quadratic in output. Solutions are the same as the standard model, but with zero new output by the exiting firm and still allowing compatibility with the exiting firm's installed base.

Aggregating output of the two remaining varieties and rearranging, total industry output is given by: $A_i + \alpha_i\,(q_i + q_j) - (q_i + q_j) + \alpha_i \sum_{k=1}^{n} b_k - c$

$$q_i + q_j = \frac{A_i + A_j + (\alpha_i + \alpha_j) \sum_{k=1}^{n} b_k - nc}{1 + n - (\alpha_i + \alpha_j)} \tag{1d}$$

That is, all parameters for remaining participants are included, but only the installed base of the exiting firm is included.

### Monopoly

If a second firm makes a negative output as a result of entry, the remaining firm monopolises the market. Inverse demand for the remaining firm is given by:

$$p_i = A_i + \alpha_i \left( s_i + \sum_{k=1}^{n+2} b_k \right) - q_i \tag{2}$$

and profit for new customers is given by:

$$\pi_i = (p_i - c)\, q_i = \left( A_i + \alpha_i \left( s_i + \sum_{k=1}^{n+2} b_k \right) - q_i - c \right) q_i \tag{2a}$$

Using the first order condition for profit maximization and correct consumer expectations for market share ($s_j = q_i$), the remaining monopolists output in Cournot competition is given by:

$$q_i = A_i + \alpha_i \left( s_i + \sum_{k=1}^{n+2} b_k \right) - q_i - c \tag{2b}$$

Substituting into profit:

$$\pi_i = (p_i - c)\, q_i = \left( A_i + \alpha_i \left( s_i + \sum_{k=1}^{n+2} b_k \right) - q_i - c \right) q_i = q_i^2 \tag{2c}$$

finds profits are still quadratic in output. Solutions are the same as the standard model, but with zero new output by the other incumbent and still allowing compatibility with the exiting incumbent's installed base. Aggregating output is not required. Rearranging output, "aggregate" output is given by:

$$q_i = \frac{A_i + \alpha_i \sum_{k=1}^{n+2} b_k - c}{1 + n - \alpha_i}. \tag{2d}$$

In calculating the aggregate output under compatibility, observe that the $A_i$ and $\alpha_i$ of the exiting firm(s) is subsequently treated as zero but the installed base is treated as usual and $n$ is revised to the number of firms with positive output.

## Autarky

### Entry, rival network exit (firm 2 or firm 3)

If a firm on the rival network to Firm 1 has negative output and if Firm 1 chooses autarky, Firm 1's inverse demand from new customers is given by:

$$p_1 = A_1 + \alpha_1 (s_1 + b_1) - q_1 - q_i \tag{3}$$

and inverse demand for new customers for the rival entrant firm is given by:

$$p_i = A_i + \alpha_i \left( s_i + \sum_{j=2}^{n+1} b_j \right) - q_1 - q_i \tag{3a}$$

Profit for Firm 1 is given by:

$$\pi_1 = (A_1 + \alpha_1 (s_1 + b_1) - q_1 - q_i - c)\, q_1 \tag{3b}$$

and for any rival firm by:

$$\pi_i = \left( A_i + \alpha_i \left( s_i + \sum_{j=2}^{n+1} b_j \right) - q_1 - q_i - c \right) q_i. \tag{3c}$$

Using the first order condition for profit maximization and correct consumer expectations for market share ($s_j = q_j$), Firm 1 and its rivals' outputs in Cournot competition are given by:

$$q_1 = \frac{A_1 + \alpha_1 b_1 - q_i - c}{(2 - \alpha_1)} \tag{3d}$$

$$q_i = A_i + \alpha_i \sum_{j=2}^{n+1} b_j - q_1 - (1 - \alpha_i)\, q_i - c \tag{3e}$$

$$q_i = A_i + \alpha_i \sum_{j=2}^{n+1} b_j - q_1 - (1 - \alpha_i)\, q_i - c \tag{3f}$$

Substituting into the profit functions:

$$\begin{aligned} \pi_1 = (A_1 + \alpha_1 b_1 - q_i - c - (1 - \alpha_1)\, q_1)\, q_1 = ((2 - \alpha_1)\, q_1 - (1 - \alpha_1)\, q_1)\, q_1 = q_1^2 \\ \pi_i = \left(A_i + \alpha_i \textstyle\sum_{j=2}^{n+1} b_j - q_1 - (1 - \alpha_i)\, q_i - c\right) q_i = q_i^2 \end{aligned} \tag{3g}$$

finds profit is again quadratic in quantities. Substituting 3d into Equation 3f and rearranging completes the equilibrium:

$$q_i = \frac{(2 - \alpha_1)\left(A_i + \alpha_i \sum_{j=2}^{n+1} b_j - c\right) - (A_1 + \alpha_1 b_1 - c)}{((2 - \alpha_1)\,(2 - \alpha_i) - 1)} \tag{3h}$$

Equations 3d, 3f and 3h define each firm's output as a function of the parameters. Observe that the calculation is equivalent to the above aggregates with $A_i$ and $\alpha_i$ of the exiting firm(s) treated as zero and $n$ revised to the number of firms with positive output.

$$\sum_{i=2}^{n+1} q_i = \frac{(2 - \alpha_1)\left(\sum_{j=2}^{n+1} A_i + \sum_{j=2}^{n+1} \alpha_i \sum_{j=2}^{n+1} b_j - (n-1)\, c\right) - (n-1)\,(A_1 + \alpha_1 b_1 - c)}{(2 - \alpha_1)\left(n - \sum_{j=2}^{n} \alpha_i\right) - (n-1)} \tag{3i}$$

**Entry, Firm 1 exit**

If Firm 1 makes negative output as a result of entry and if Firm 1 chooses autarky, output is set to zero ($q_1 = 0$) and inverse demand for new customers for the rival firms is given by:

$$p_i = A_i + \alpha_i \left(s_i + s_j + \sum_{j=2}^{n+1} b_j\right) - q_i - q_j \tag{4}$$

Profit for any rival firm is given by:

$$\pi_i = \left(A_i + \alpha_i \left(s_i + s_j + \sum_{j=2}^{n+1} b_j\right) - q_i - q_j - c\right) q_i. \tag{4a}$$

Using the first order condition:

$$q_i = A_i + (\alpha_i - 1)\,(q_i + q_j) + \alpha_i \sum_{j=2}^{n+1} b_j - c \tag{4b}$$

Substituting into profit finds profits are quadratic in quantities. Aggregating:

$$(q_i + q_j) = \frac{\sum_{i=2}^{n+1} A_i + \sum_{i=2}^{n+1} \alpha_i \sum_{j=2}^{n+1} b_j - nc}{\left(1 + n - \sum_{i=2}^{n+1} \alpha_i\right)}. \tag{4c}$$

As $q_1$ is forced to be zero, this is all that is required. Observe the similarity with aggregate output

**Autarkic monopoly**

If the remaining rival firm has a negative output and Firm 1 chooses Autarky, Firm 1's inverse demand from new customers is given by:

$$p_1 = A_1 + \alpha_1\,(s_1 + b_1) - q_1 \tag{5}$$

Profit for Firm 1 is given by:

$$\pi_1 = (A_1 + \alpha_1\,(s_1 + b_1) - q_1 - c)\, q_1. \tag{5a}$$

Using the first order condition for profit maximization and correct consumer expectations for market share $(s_j = q_j)$, Firm 1's output in Cournot competition is given by:

$$q_1 = \frac{A_1 + \alpha_1 b_1 - c}{(2 - \alpha_1)} \tag{5b}$$

No aggregate is required.

**Rival network monopoly**

If the Firm 1 subsequently has negative output, but still chooses Autarky the market is monopolised by the remaining Firm on the rival network. This scenario is possible if Firm 2 acquires the innovation. Although it is unlikely that Firm 1 would choose Autarky, this theoretical scenario is required for a sensitivity analysis and for comparing bids under all possible scenarios.

If Firm 1 makes negative output as a result of entry and if Firm 1 chooses autarky, output is set to zero $(q_1 = 0)$ and inverse demand for new customers for the rival firms is given by:

$$p_i = A_i + \alpha_i \left( s_i + \sum_{j=2}^{n+1} b_j \right) - q_i \tag{6}$$

Profit for any rival firm is given by:

$$\pi_i = \left( A_i + \alpha_i \left( s_i + \sum_{j=2}^{n+2} b_j \right) - q_i - c \right) q_i. \tag{6a}$$

Using the first order condition:

$$q_i = A_i + (\alpha_i - 1)\, q_i + \alpha_i \sum_{j=2}^{n+2} b_j - c \tag{6b}$$

Substituting into profit finds profits are quadratic in quantities. Rearranging further:

$$q_i = \frac{A_i + \alpha_i \sum_{j=2}^{n+2} b_j - nc}{(2 - \alpha_i)}. \tag{6c}$$